

11991

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e)
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x) = 24x - 18$

(ii) $f'(1) = -6$

(iii) $f(2) = 0$

(a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.

(b) Write an expression for $f(x)$.

(c) Find the average value of f on the interval $1 \leq x \leq 3$.

2. Let R be the region between the graphs of $y = 1 + \sin(\pi x)$ and $y = x^2$ from $x = 0$ to $x = 1$.

(a) Find the area of R .

(b) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x -axis.

(c) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y -axis.

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3. Let f be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$.
- Write an equation for the line tangent to the graph of f at the point where $x = 0$.
 - Using the equation found in part (a), approximate $f(0.02)$.
 - Let f^{-1} denote the inverse function of f . Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1} .

4. Let f be the function given by $f(x) = \frac{|x| - 2}{x - 2}$.

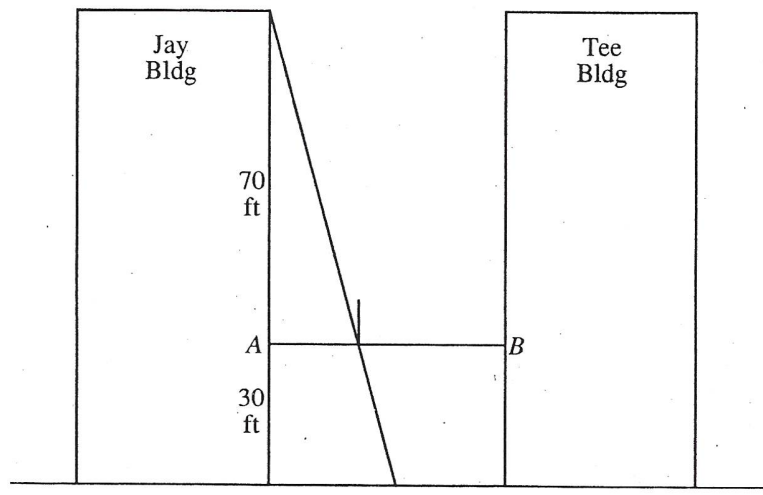
- Find all the zeros of f .
- Find $f'(1)$.
- Find $f'(-1)$.
- Find the range of f .

5. Let f be a function that is even and continuous on the closed interval $[-3, 3]$. The function f and its derivatives have the properties indicated in the table below. *symmetric about y axis*

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

- Find the x -coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x -coordinate you give, state whether f attains an absolute maximum or an absolute minimum.
- Find the x -coordinate of each point of inflection on the graph of f . Justify your answer.
- In the xy -plane provided below, sketch the graph of a function with all the given characteristics of f .

Note: The xy -plane is provided in the pink test booklet only.



6. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B , is illuminated by a spotlight 70 feet above point A , as shown in the diagram.
- How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)
 - How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)
 - How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee building when she is 10 feet from point B ? (Indicate units of measure.)

END OF EXAMINATION.